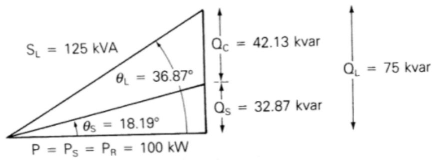
Solution to Assignment 1

QUESTION 1 (10 marks)

The power triangle is:



After adding the capacitor, the power angle becomes

$\theta_S = \delta - \beta_S = \cos^{-1}(0.95) = 18.19°$

and the reactive power and apparent power supplied by the source are

$Q_S = P \tan \theta_S = 100 \tan(18.19°) = 32.87$ kvar

$S_S = P / (\cos \theta_S) = 100 / 0.95 = 105.6$ kVA

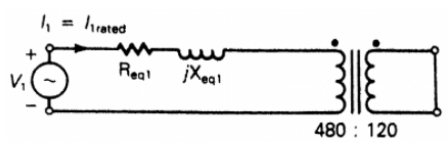
Therefore, the capacitor contributes

$Q_C = Q_L - Q_S = 75 - 32.87 = 42.13$ kvar

Note: The apparent power from the source was originally 125 kVA (purely due to the inductor), and after adding the capacitor, the apparent power from the source drops to 105.6 kVA.

QUESTION 2 (20 marks)

(a) From the equivalent circuit for the short-circuit test:



we can find the rated current for winding 1 as

$I_{rated} = S_{rated} / V_{rated} = 20000/480 = 41.667$ A

Therefore,

$R_{eq1} = P_1 / I_{rated}^2 = 300/(41.667^2) = 0.1728\ \Omega$

$|Z_{eq1}| = V_1 / I_{1rated} = 35/41.667 = 0.84\ \Omega$

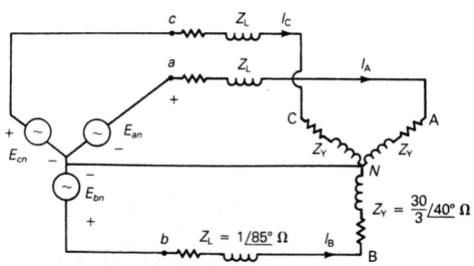
$X_{eq1} = \sqrt{(Z_{eq1}^2 - R_{eq1}^2)} = 0.8220\ \Omega$

So,

$Z_{eq1} = R_{eq1} + j X_{eq1} = 0.1728 + j\,0.8220 = 0.84\angle 78.13°\ \Omega$

QUESTION 3 (20 marks)

The Δ-load circuit is first converted to a Y-load circuit. Since the circuit is **balanced**, the neutral points can be optionally connected. The circuit diagram is



The line currents are

$I_A = E_{an} / (Z_L + Z_Y) = \dfrac{\frac{480}{\sqrt{3}}\angle{-30^o}}{1\angle 85^o + \frac{30}{3}\angle 40^o} = 25.83\angle{-73.78^o}$ A

$I_B = 25.83\angle 166.22^o$ A (which is 120° lagging I_A)

$I_C = 25.83\angle 46.22^o$ A (which is 120° lagging I_B)

Now we can translate them back to the Δ-load currents.

$I_{AB} = (I_A/\sqrt{3})\angle{+30^o} = 14.91\angle{-43.78^o}$ A

$I_{BC} = 14.91\angle{-163.78^o}$ A

$I_{CA} = 14.91\angle{+76.22^o}$ A

The voltages across the load terminals are:

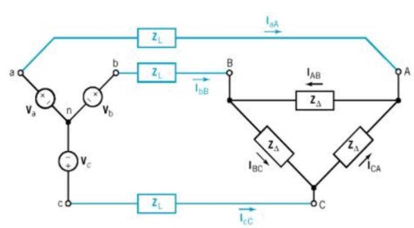
$V_{AB} = Z_\Delta I_{AB} = (30\angle 40^o)(14.91\angle{-43.78^o}) = 447.3\angle{-3.78^o}$ V

$V_{BC} = 447.3\angle{-123.78^o}$ V

$V_{CA} = 447.3\angle 116.22^o$ V

QUESTION 4 (20 marks)

First, replace the Δ-load by Y-load so that a single-phase equivalent can be used.



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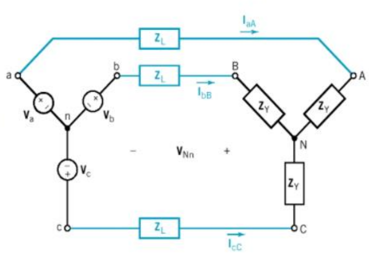
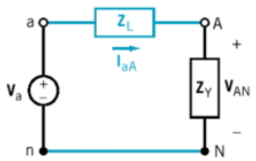


Here, $Z_Y = Z_\Delta/3 = 25 + j\,75\ \Omega$.

The single-phase equivalent is:



$I_{aA} = V_a / (Z_L + Z_Y) = 1.26\angle{-66^\circ}$ A (rms)

$I_{bB} = 1.26\angle{-186^\circ}$ A (rms)

$I_{cC} = 1.26\angle{-54^\circ}$ A (rms)

The line-to-neutral voltages at load are therefore:

$V_{AN} = I_{aA}Z_Y = 99.6\angle{5^\circ}$ V (rms)

$V_{BN} = 99.6\angle{-115^\circ}$ V (rms)

$V_{CN} = 99.6\angle{+125^\circ}$ V (rms)

The line-to-line voltages at load are:

$V_{AB} = V_{AN} - V_{BN} = 172\angle{35^\circ}$ V (rms) or simply use $\sqrt{3}V_{AN}$ and shifting 30°

$V_{BC} = 172\angle{-85^\circ}$ V (rms)

$V_{CA} = 172\angle{155^\circ}$ V (rms)

The current in the each Δ-load is

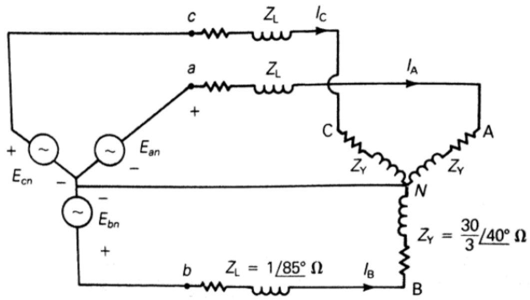
$I_{AB} = V_{AB} / Z_\Delta = 0.727\angle{-36^\circ}$ A (rms)

$I_{BC} = V_{BC} / Z_\Delta = 0.727\angle{-156^\circ}$ A (rms)

$I_{CA} = V_{CA} / Z_\Delta = 0.727\angle{84^\circ}$ A (rms)

QUESTION 5 (20 marks)

Convert the Δ-load to Y-load.



The base impedance is

$Z_{base} = V_{baseLL}{}^2/S_{base3\phi} = 480^2/10000 = 23.04\ \Omega$

The per-unit line and load impedances are

$Z_{Lpu} = Z_L / Z_{base} = 1\angle 85^o / 23.04 = 0.0434\angle 85^o\ \Omega$

$Z_{Ypu} = Z_Y / Z_{base} = 10\angle 40^o / 23.04 = 0.434\angle 40^o\ \Omega$

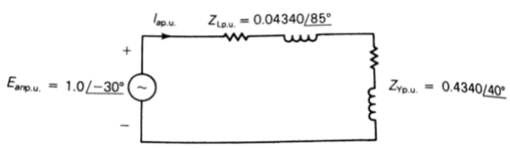
Then, the line-to-neutral base voltage is

$V_{baseLN} = V_{baseLL}/\sqrt{3} = 480/\sqrt{3} = 277$ V

Thus, the per-unit line-to-neutral voltage of source is

$E_{anpu} = E_{an} / V_{baseLN} = 277\angle -30^o / 277 = 1\angle -30^o$ pu

The single-phase equivalent circuit is:



$I_{anpu} = E_{anpu} / (Z_{Lpu} + Z_{Ypu}) = 1\angle -30^o / [\,0.0434\angle 85^o + 0.434\angle 40\,] = 2.147\angle -73.78^o$ pu

Since the base current is Ibase $= S_{base3\phi} / \sqrt{3}\, V_{baseLL} = 10000/(\sqrt{3} \times 480) = 12.03$ A, we have

$I_a = I_{anpu} \times 12.03 = 25.83\angle -73.78^o$ A

QUESTION 6 (10 marks)

Power supplied to the 3-phase load = W1 + W2 = 5.8 kW.

Also, $\tan\phi = \sqrt{3}(W2-W1) / (W2+W1) = \sqrt{3} \times 0.4 / 5.8 = 0.11945$

Thus, $\phi = 6.8^o$.

Suppose the load is $Z\angle\phi\ \Omega$.

Active power 5.8 kW/3 $= (220^2 / Z) \cos(6.8^o) \Longrightarrow Z = 24.86\ \Omega$

Therefore, the load impedance is $24.86\ \angle 6.8^o\ \Omega$ OR $24.68 + j2.94\ \Omega$

$$P = \frac{3V_{\text{ph}}^2}{2|Z|} = \frac{3V_{\text{ph,rms}}^2}{|Z|}$$

Here, the power should be divided by 3 first then to get the each load impedance in Y.